Nasim ROUDABR, PhD Candidate
E-mail: nasim.roudabr@srbiau.ac.ir
Department of Industrial Engineering
Science and Research Branch, Islamic Azad University, Tehran, Iran
Assistant Professor Seyyed Esmaeil NAJAFI, PhD
E-mail: e.najafi@srbiau.ac.ir
Department of Industrial Engineering
Science and Research Branch, Islamic Azad University, Tehran, Iran
Assistant Professor Zohreh MOGHADDAS, PhD
E-mail: Zmoghaddas@qiau.ac.ir
Department of Mathematics and Statistics
Qazvin Branch, Islamic Azad University, Qazvin, Iran
Assistant Professor Farzad Movahedi SOBHANI, PhD
E-mail: Fmovahedi@iau.ac.ir
Department of Industrial Engineering
Science and Research Branch, Islamic Azad University, Tehran, Iran

A NEW MODELING APPROACH FOR UNDESIRABLE FACTORS IN EFFICIENCY EVALUATION OF CEMENT INDUSTRY WITH FOUR STAGES STRUCTURE BASED ON PIECEWISE LINEAR NDEA MODEL

Abstract. *The undesirable outputs have a negative impact on the efficiency of DMUs and will be unavoidable. So, a method for evaluating efficiency should be able to consider nonlinear value of undesirable outputs and improve efficiency of DMUs with decreasing undesirable outputs and increasing desirable ones. Network data envelopment analysis (NDEA) models consider the internal structure of a production system while measuring efficiencies, But are based on the assumption that the weights coupled with the ratio scales of the inputs and outputs imply linear value functions. Linear value may not properly show the differences in the value obtained from one DMU to another. So, this paper propose a new model according to the NDEA technique and a piecewise linear function, which considers the effects of undesirable outputs in efficiency evaluation and distinguishes the valuation between different amount of undesirable outputs have produced by different DMUs. The proposed model was employed to evaluate efficiency of the 42 cement companies.*

Keywords*: efficiency, piecewise linear function, network DEA, undesirable outputs, cement Industry.*

JEL Classification: C02, C44, C61, C67.

DOI: 10.24818/18423264/56.1.22.04

1. Introduction

In conventional DEA models, DMUs are regarded as black-boxes, whereas the internal structure and operation of DMUs are ignored. Therefore, they cannot provide information on the performance of internal stages, and the resultant efficiency can be misleading. Hence, the network DEA (proposed by Fare and Grosskopf, 2000), considering the internal processes of systems, can determine the relative performance of network structure systems and yield more meaningful and informative results than the traditional DEA. There are hundreds of studies on the network DEA. Some of them have developed specific models to measure efficiency under special conditions, whereas some others have analyzed the properties of certain models. Other studies have used the existing models to solve real-world problems (Kao, 2014).

The production of undesirable outputs such as waste, pollution, aerosols, and others alongside desirable outputs is one of the main reasons for inefficient production processes. The production of undesirable outputs that have a negative impact on DMUs efficiency will be unavoidable and the presence of such outputs has a significant effect on estimation of DMUs efficiency. In such conditions, the designated efficiency evaluation method should be able to decrease undesirable outputs and increase desirable ones while being compatible with the concepts of production theory. D'Inverno *et al.* (2018) considered the undesirable output and performed a new integrated analytic hierarchy process, *i.e.* a directional non-radial distance function. Huang and Chung (2017) considered a directional technology distance function so that the desirable factors would be expanded and that the undesirable ones would be contracted simultaneously. Khoshroo *et al.* (2018) formulated a new non-radial DEA-based efficiency model for efficiency analysis. Song *et al.* (2018) developed an improved approach in order to assess resources and environmental efficiency based on DEA. Their approach introduced the assessment of resource inputs into an objective function. The new model can measure efficiency in regard to resource input, undesirable outputs, and desirable outputs. Liu *et al.* (2015) analyzed two-stage DEA models with undesirable input-intermediate-outputs. They employed the free disposable hull to create production possibility sets (PPS) and the corresponding DEA models with undesirable variables. Kalhor and Matin (2018) modeled a general network DEA approach in the presence of undesirable outputs. They proposed some network DEA models with undesirable outputs to evaluate the performance of production units. Maleki *et al.* (2019) introduced a novel PPS for a two-stage network in the presence of undesirable intermediate products and nondiscretionary exogenous inputs.

Fare and Grosskopf (2000) suggested the necessity of adopting the NDEA model, which had been used in many areas (cf. Cook's (2010) review paper). The NDEA modelling technique was also employed in this paper. In addition, since a large number of real-world problems entail different inputs and outputs with nonlinear values, the results will not be accurate, reliable, and useful if linear valuation is used in performance evaluation (values are the coefficients of inputs or outputs in modelling and can be the price of desirable outputs or the cost of inputs and undesirable outputs). The piecewise linear function was hence employed to distinctly valuate the variables of a DMU with those of another (Cook & Zhu, 2009). Seiford and Zhu (2002) reckon if undesirable outputs are used as inputs, then the DEA results will not accurately represent the production process. Therefore, undesirable outputs should be employed in modelling so that as it increases the value that the model associates with, the scale should decrease. In other words, undesirable and desirable output valuation should be differentiated. Since the DEA models are intended to increase output, however, they are unable to distinguish between desirable and undesirable outputs and will associate a higher value to undesirable outputs. For instance, highly polluted air will people more allergic, therefore different degrees of importance should be attributed to different levels of pollution. In efficiency measurements, it is essential to attribute different levels of importance to different values of undesirable outputs. Hence, in order to analyze the effects of undesirable outputs on performance evaluation and obtain accurate and useful results, the nonlinear valuation and behavior of outputs should also be considered in modelling.

This paper aimed to develop and correct certain shortcomings in performance evaluation models that don't produce useful and accurate results due to undesirable outputs. Compared to previous studies, this paper has implemented the following innovations:

- Proposing a new idea for considering the effects of undesirable outputs on performance evaluation while taking different weights into account in order to justify the nonlinear value of undesirable outputs.
- Developing a mathematical model with the piecewise linear function to account for undesirable outputs in performance evaluation.
- Since undesirable outputs (e.g. greenhouse gas emissions) significantly impact the efficiency of cement companies in production and consumption processes, this paper proposed a general network structure designed through NDEA modelling for the first time to evaluate the efficiency of cement companies by analyzing the nonlinear value of undesirable outputs in the modelling process.

The rest of the paper consists of the following sections. NDEA model is presented along with the piecewise linear DEA in Section 2. Section 3 discusses the proposed model for measuring efficiency with piecewise linear factors in NDEA. The model is employed in Section 4 to evaluate the efficiency of 42 cement facilities in Iran. The empirical results are also presented in this section. The conclusion is finally drawn in Section 5.

2. Preliminaries

2.1. Network Data Envelopment Analysis (NDEA)

In the conventional DEA, DMUs are usually formulated as a single process transforming inputs into outputs. They are treated as a black-box in which internal structures are generally ignored (Fare and Grosskopf, 2000). For the systems consisting of some interconnected processes, the above models ignore the performance of sub-processes. Thus, the NDEA is employed to consider DMUs with complex internal structures. The NDEA consists of two basic structures, named serial and parallel (Kao, 2014). For the sake of simplicity, a serial structure is considered with a two-stage process according to Fig. 1. Based on the input-oriented DEA model proposed by Charnes *et al.* (1978); Kao and Hwang (2008) presented a new model to measure the overall efficiency, which can be transformed into the following linear program:

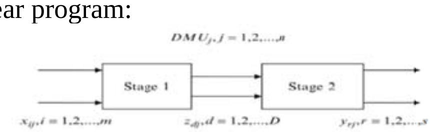

Figure 1. Two stages process

Similarly, it is possible to develop output-oriented models. Model (1) yields the overall efficiency. After the overall efficiency is obtained, divisional efficiency can be obtained through efficiency decomposition (Kao & Hwang, (2008)). Model (1) is the multiplier CCR model, the dual of which can be formulated its dual as follows (2):

(1)

$$\begin{aligned}
\max\ & \sum_{r=1}^{s} u_r y_{ro} \\
S.t:\ & \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0; \quad j = 1,\dots,n, \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0; \quad j = 1,\dots,n, \\
& u_r, w_d, v_i \ge 0; \quad r = 1,\dots,s;\ \ d = 1,\dots,D;\ \ i = 1,\dots,m.
\end{aligned}$$

(2)

$$\begin{aligned}
\min\ & \theta \\
s.t:\ & \\
& \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{io}; \quad i = 1,\dots,m;\ \ j = 1,\dots,n, \\
& \sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{ro}; \quad r = 1,\dots,s;\ \ j = 1,\dots,n, \\
& \sum_{j=1}^{n} \lambda_j^1 z_{dj} \ge \sum_{j=1}^{n} \lambda_j^2 z_{dj}; \quad d = 1,\dots,D;\ \ j = 1,\dots,n, \\
& \lambda_j^1 \ge 0; \quad \lambda_j^2 \ge 0; \quad j = 1,\dots,n.
\end{aligned}$$

In many cases, DMUs may have network structures. These kinds of DMUs have inputs, outputs, and intermediate amounts flowing from one stage to another. The stages may also have its own inputs and outputs. Kao (2017) stated that the NDEA was a relatively new subject with a short history of no more than 20 years since the term first appeared in 2000. In the last two decades, dozens of models have been proposed, whereas new models are still being developed. In all of the studies in which NDEA models were employed, the aggregate output–input was assigned as a linear function of each output–input. However, linear value cannot reveal the reality of situations in which there are variables leaving nonlinear impacts on the efficiency measures of complex systems.

2.2. Piecewise Linear models in DEA (PLDEA)

Cook and Zhu (2009) stated that, in the standard DEA model, the aggregate output (input) was a pure linear function of each output (input). This means that if DMU_{j_1} generates twice as much of an output as another DMU_{j_2} does, then DMU_{j_1} is credited with having created twice as much value. In many situations, however, linear pricing ($\mu_r y_{rj}$) may not sufficiently show differences in values obtained from one DMU to another. Therefore, they solved this problem and proposed the PLDEA. They established that certain factors, previously treated as behaving linearly, should be looked upon as having a nonlinear effect on efficiency. Thus, they have considered the theory of piecewise linear programming, when the scale of variable with nonlinear effect could be divided into k segments with each variable supposed to behave linearly in those segments. Clearly, the more the segments, the closer the piecewise linear estimation to the actual nonlinear function. Accordingly, the scale of the variable indicating the diminishing marginal value (DMV) behavior should be divided into k_r ranges $[0,L_1]$, $(L_1,L_2],\ldots,(L_{k_r-1},L_{k_r}]$ .Let u_{r_k} be the value assigned to the portion of y_{rj} which lies within the k*th* range (Hosseinzadeh Lotfi et al, 2010).

If $y_{rj} \in (L_{k_j-1}, L_{k_j}]$ then the parameters y_{rj}^k are defined as follows:

$$Y_{rj}^k = \begin{cases} L_K, & if\ K = 1, \\ L_K - L_{K-1}, & if\ K = 2,\ldots,K_j - 1, \\ Y_{rj} - L_{K-1}, & if\ K = K_j, \\ 0, & if\ K > K_j. \end{cases} \tag{3}$$

The PLDEA model developed by Cook and Zhu (2009) is as follows:

$$\begin{aligned} \max\ & \sum_{r\in R_1} u_r y_{ro} + \sum_{r\in R_2} \sum_{k=1}^{K_r} u_{r_k} y_{ro}^k, \\ s.t.\ & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r\in R_1} u_r y_{rj} + \sum_{r\in R_2} \sum_{k=1}^{K_r} u_{r_k} y_{rj}^k - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,\ldots,n, \\ & u_{r_{k-1}} a_{r_k} \le u_{r_k} \le u_{r_{k-1}} b_{r_k}, \quad k = 1,\ldots,K_r, \quad r \in R_2, \qquad (a) \\ & u_{r_1} a_{r_1 r_2} y_{r_2 j} \le \sum_{k=1}^{K_{r_2}} u_{r_{2k}} y_{r_2 j}^k \le u_{r_1} b_{r_1 r_2} y_{r_2 j}, \quad j = 1,\ldots,n, \quad r_1 \in R_1, \quad r_2 \in R_2, \quad (b) \\ & v_i, u_r, u_{r_k} \ge 0. \end{aligned} \tag{4}$$

In Model (4), R_1 and R_2 are employed to define sets of regular and DMV outputs , J={1,…,n} and $r_1 \in R_1$, $r_2 \in R_2$, respectively. Cook and Zhu (2009) showed that $f(y_{rj}) = u_r(j) y_{rj}$ was the linear equivalent of the piecewise linear function $\sum_{k=1}^{K} u_{r_k} y_{rj}^k$, where $u_r(j)$ is a convex combination of $\{u_{r_k}\}_{k=1}^{K}$. It should be stated that a_{r_k} and b_{r_k} take values strictly greater than one for the mentioned variable. The parameters $a_{r_1 r_2}$ and $b_{r_1 r_2}$ are the lower and upper bounds on the ratios of pairs of regular and mentioned variables, respectively. The choice of number, width of ranges, and bounds on the ratios of pairs of variables need to be carefully determined by an analyst.

Hosseinzadeh Lotfi *et al.* (2010) showed that the model proposed by Cook and Zhu (2009) failed to produce acceptable targets. Thus, they improved the piecewise linear CCR model to facilitate producing Pareto-efficient targets. They also applied non-radial improvements to fill the lower ranges before filling the upper ones (see Hosseinzadeh Lotfi et al. (2010)).

Ji *et al.* (2018) combined the DEA algorithm with classification information and presented a novel DEA-based classifier to construct a piecewise linear discriminant function. In this classifier, class information is added, and the nonnegative conditions of DEA model are lost.

3. The proposed model

In many studies, network production systems are restricted to have serial or parallel structure. Fukuyama and Weber (2010) employed the slack-based inefficiency (SBI) and directional distance function (DDF) models to determine an efficiency score of DMUs with undesirable outputs in the general structure of NDEA. Lozano et al. (2013) analyzed the general NDEA with undesirable outputs. They proposed a production technology for this structure and also introduced a DDF model for performance evaluation. Kalhor and Kazemi Matin (2018) modeled a general NDEA, considered in the presence of undesirable outputs. Some NDEA models were also developed for performance evaluation of production units.

Few papers have evaluated the efficiency of a general network structure with respect to undesirable outputs while suggesting that considering undesirable outputs in a general network structure could affect efficiency and make the results more accurate. This paper aims to address the following question: “which model can properly account for the effects of undesirable outputs on a general network structure and determine efficiency?” As mentioned earlier, this paper is intended for developing a DEA-based model to evaluate sustainable performance in different industries. The cement industry has a strategic and significant role in a nation’s civil and economic development and is characterized by the largest production rate of all other industries in the modern era. Some argue that cement production is an indicator of growth and development for every country, and has a key role as a fundamental industry in developing Iran’s economic infrastructures. So, in this section we consider a general network structure and propose a new model based on NDEA to evaluate efficiency of DMUs in the presence of piecewise linear undesirable outputs. The proposed model was then validated by evaluating the efficiency of Iranian cement companies in 2016. Fig. 2 presents the proposed model framework.

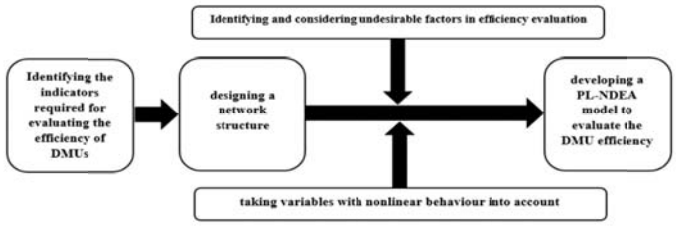

Figure 2. The framework of proposed model

According to Figure 2, the proposed model consists of five main stages. 1) Identifying the performance indicators used for evaluating DMU efficiency. In this paper, the most important performance indicators of companies were selected from the literature according to expert opinion. 2) Designing the network structure after specifying the final indicators. 3) As previously discussed in detail, identifying and determining undesirable outputs is an important step in using DEA models to evaluate DMU efficiency. 4) Since most real-world problems entail specific inputs and outputs with nonlinear values, the nonlinear behaviour and valuation of variables in the modelling process should be tailored for distinguishing DMUs in terms of valuation. 5) Finally, a new model was proposed for the efficiency evaluation of general network structures according to undesirable outputs. Their nonlinear effect was assessed by using the NDEA technique and the piecewise linear function.

As discussed earlier, selecting input and output variables is an important step in evaluating the efficiency of industrial DMUs with the DEA technique. In other words, the incorrect selection of required variables will invalidate the evaluation results of cement industries. According to the collection and accessibility of data on research variables, this study employed variables from previous studies regarding the performance evaluation of global cement companies and expert opinions in order to select the variables of inputs and outputs demonstrated in Table1. The sustainability of the cement industry depends on operational, managerial, and technical factors. Since environmental problems have increasingly drawn the public’s attention in the past three decades, identifying the indicators of environmental sustainability is an important and unavoidable step in evaluation. The identification necessitates the detailed evaluation of quality and the impact of each indicator.

The key performance indicators were selected according to four important levels, namely environmental sustainability, strategic, process, and operational. they are as follows: **X:** (1) quality of suppliers in terms of sustainable supply of minerals and consumables, (2) cost of green and sustainability education for following relevant rules in the supply chain, (3) total initial investment in mine exploitation and factory processes, (4) total salaries and wages, (5) total costs of purchasing minerals, chemicals, and other consumable substances, (6) the sum of money paid to contractors for mining, (7) total transportation cost, (8) total financial costs, (9) total number of employees, (10) total debts of factories. **Z:** (1) total available mineral reserves, (2) total tonnage of extracted minerals, (3) total tonnage of chemical and mineral raw materials added to the production process, (4) total mineral raw materials in storage depots for use in cold seasons, (5) the quality of training programs for suppliers and employees for sustainable production and TQM, (6) annual Mazut energy fuel consumption in lit/t, (7) total research and development expenditures, (8) total energy cost, (9) actual industry capacity, (10) annual power consumption in kw/h, (11) annual gas consumption in m3/t, (12) the environmentally degrading effects of mining. **M:** (1) response cost of Total dust particles produced in Mg/Nm3, (2) response cost of average annual greenhouse gas emissions of NOX in Mg/Nm3, (3) response cost of average annual greenhouse gas emissions of SO2 in Mg/Nm3, (4) response cost of average annual greenhouse gas emissions of CO in Mg/Nm3. **K:** (1) the factory's total cement production tonnage, (2) the factory's total clinker production tonnage, (3) the total impact of the permeation of drinking water and wastewater into groundwater, (4) average annual greenhouse gas emissions of NOX in Mg/Nm3, (5) average annual greenhouse gas emissions of SO2 in Mg/Nm3, (6) average annual greenhouse gas emissions of CO in Mg/Nm3, (7) total dust particles produced in Mg/Nm3. **z':** (1) suppliers' flexibility, (2) improving the relationships in the supply chain, (3) the total cost of increasing supply chain reliability, (4) compliance with legal standards and governmental rules in the supply chain. **L:** (1) total marketing costs, (2) warehouse inventory value in rial (materials and goods), (3) total tonnage of bagged and bulk cement sold in domestic and export markets, (4) total tonnage of clinker sold, (5) number of pp cement bags consumed annually, (6) consumer price. **K':** (1) inverse procurements, (2) the effect of factory efficiency on creating negative conditions in the ecosystem, (3) the costs of environmentally friendly design, (4) social responsibility, (5) efforts for using advanced technologies and substitute raw materials. **Y:** (1) total current assets, (2) factory brand competitiveness and globalization, (3) green space development culture, (4) total earnings from product sales, (5) total profits, (6) annual growth rate according to performance, (7) return on assets (ROA), (8) return on equity (ROE), (9) effectiveness of factory in a particular area of activity, (10) customer satisfaction, (11) implementing quality working conditions for personnel, (12) social responsibility, (13) ton of waste (bags) disposed in the environment, (14) ton of pollution caused by emitting unrecyclable substances.

It is important to determine inputs and outputs in implementing the NDEA efficiency evaluation model on which there is commonly no consensus. Inputs and outputs are usually determined on the basis of their common definitions. In other words, an input variable is one that increases efficiency as it decreases, whereas an output variable is one which increases efficiency as it increases In addition, the presence of undesirable outputs such as dust emission, greenhouse gases, unrecyclable substances, and waste disposal can increase inefficiency or negatively impact the cement industry's efficiency, which should be taken into account in modelling and designing a network structure.

For a better understanding of proposed method, we consider information of the 42 cement firms in Iran for year 2016 has been Extracted from Tehran Stock Exchange. We determine inputs, desirable, and undesirable outputs for each stage. The general network structure includes 42 homogenous DMUs, all of which consist of four stages and also have similar internal structures and internal relations presented Figure 3. According to Fig. 3, X is set of inputs consumed by Stage 1, whereas Z is a set of intermediate products consumed by Stage 2 and produced by Stage 1. Furthermore, Z' is set of intermediate products consumed by Stage 3 and produced by Stage 1, whereas M is a set of inputs consumed by Stage 2, and K is set of intermediate products consumed by Stage 3 and produced by Stage 2. In addition, K' is a set of intermediate products consumed by Stage 4 and produced by Stage 2, whereas L is a set of intermediate products consumed by Stage 4 and produced by Stage 3, and Y is a set of outputs produced by Stage 4.

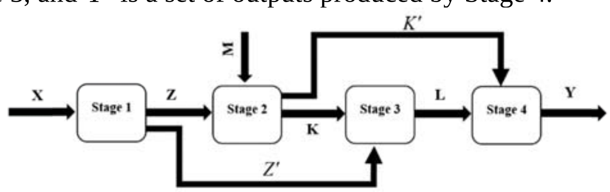

Figure 3. the proposed network structure

Hence, a new model was proposed for efficiency evaluation of general network structures by identifying indicators according to undesirable outputs, the suitable corresponding network structure, the NDEA technique, and a piecewise linear function. According to Seiford and Zhu (2002) if undesirable outputs are used as input, the DEA results will not reflect the production process; therefore, undesirable outputs should be accommodated in modelling in a way that increasing their scale decreases the value associated to them by the model; in other words, undesirable outputs should be valued separately from desirable ones. However, since DEA models seek to increase outputs, they are unable to distinguish between desirable and undesirable outputs and attribute higher values to the larger scales of undesirable outputs. Since changing the scales of undesirable outputs will impact efficiency differently, it is necessary to account for the nonlinear valuation of outputs in modelling in order to analyze the effects of undesirable outputs on efficiency and achieve accurate and useful results. To this end, a piecewise linear function was employed mainly to consider the value of each undesirable output according to its output scale, so the higher the undesirable output, the lower its value. This paper's innovation regarding the evaluation of general network structure efficiency through undesirable outputs is that for the first time, it considered the value of each undesirable output as equal to its inverse scale in the general network structure.

A piecewise linear function was used for this purpose. In other words, the higher the scales of undesirable outputs, the lower their corresponding values (the cone ratio, proposed by Charnes *et al.* (1990) was therefore used in weight constraints). As a result, a company that produces undesirable outputs on a large scale in the cement production process will be less efficient. This model can therefore result in accurate, and useful results for strategic decision-making. To better perceive the modelling process, consider the information on DMU_1: K'=(3, 2.33, 11276.05, 2.89, 3.78); K=(319753, 383703, 2.56, 476.89, 2200, 700, 130); L=(13576.36, 148732, 319433.25, 0, 7346964.68, 225521); M=(20.38, 0, 1136.17, 151.81); X=(3.67, 3743.65, 88000, 51418.79, 22461.89, 27920.15, 43976.60, 27203, 80, 351205); Y=(225706, 3.67, 3.33, 249373, 1369, 5, 0.02, 0.06, 3, 2.44, 3.22, 3.33, 331.02, 191.21); Z=(31449600, 569798.96, 17918.93, 98280, 3.78, 763568.97, 20071.37, 51869.83, 393120, 41052383.97, 35983667.34, 2.89); Z'=(3.56, 4, 13531.26, 3.33).

Therefore, the modelling process to evaluate the efficiency of network-structured systems is carried out using a piecewise linear function while considering undesirable outputs in the following manner:

$$\max \sum_{r_Y=1}^{14} u_{r_Y}^{4} Y_{r_Y j}^{4}$$

$$S.t$$

$$v^1 X_o^1 + v^2 M_o^2 = 1$$

$$-v^1 X_j^1 - v^2 M_j^2 + \sum_{r_Y=1}^{14} u_{r_Y}^{4} Y_{r_Y j}^{4} - p^{1\,2} Z_j^{1\,2} + \sum_{r_Z=1}^{12} u_{r_Z}^{1\,2} Z_{r_Z j}^{1\,2} - v^{1\,3} Z_j^{\prime 1\,3} + u^{1\,3} Z_j^{\prime 1\,3} - p^{2\,4} K_j^{\prime 2\,4} +$$

$$u^{2\,4} K_j^{\prime 2\,4} - p^{3\,4} L_j^{3\,4} + u^{3\,4} L_j^{3\,4} - p^{2\,3} K_j^{2\,3} + \sum_{r_K=1}^{7} u_{r_K}^{2\,3} K_{r_K j}^{2\,3} \le 0 \qquad j = 1, \dots, 42 \qquad (a)$$

$$-\sum_{r_Z=1}^{12} u_{r_Z}^{1\,2} Z_{r_Z j}^{1\,2} + u^{1\,3} Z_j^{\prime 1\,3} - v^1 X_j^1 \le 0 \qquad j = 1, \dots, 42 \qquad (b)$$

$$u^{2\,4} K_j^{\prime 2\,4} - \sum_{r_K=1}^{7} u_{r_K}^{2\,3} K_{r_K j}^{2\,3} - v^2 M_j^2 - p^{1\,2} Z_j^{1\,2} \le 0 \qquad j = 1, \dots, 42 \qquad (c)$$

$$u^{3\,4} L_j^{3\,4} - p^{2\,3} K_j^{2\,3} - p^{1\,3} Z_j^{\prime 1\,3} \le 0 \qquad j = 1, \dots, 42 \qquad (d)$$

$$\sum_{r_Y=1}^{14} u_{r_Y}^{4} Y_{r_Y j}^{4} - p^{2\,4} K_j^{\prime 2\,4} - p^{3\,4} L_j^{3\,4} \le 0 \qquad j = 1, \dots, 42 \qquad (e)$$

$$\left.\begin{aligned}
&\sum_{r_Y=1}^{14} u_{r_Y}^{4} Y_{r_Y j}^{4} = \sum_{r_Y=1}^{12} u_{r_Y}^{4} Y_{r_Y j}^{4} + \sum_{K_{1,r_Y}=1}^{3} u_{13}^{4,K_{1,r_Y}} Y_{13j}^{4,K_{1,r_Y}} + \sum_{K_{2,r_Y}=1}^{4} u_{14}^{4,K_{2,r_Y}} Y_{14j}^{4,K_{2,r_Y}} \\
&\sum_{r_Z=1}^{12} u_{r_Z}^{1\,2} Z_{r_Z j}^{1\,2} = \sum_{r_Z=1}^{11} u_{r_Z}^{1\,2} Z_{r_Z j}^{1\,2} + \sum_{K_{r_Z}=1}^{3} u_{12}^{1\,2,K_{r_Z}} Z_{12j}^{1\,2,K_{r_Z}} \\
&\sum_{r_K=1}^{7} u_{r_K}^{2\,3} K_{r_K j}^{2\,3} = \sum_{r_K=1}^{3} u_{r_K}^{2\,3} K_{r_K j}^{2\,3} + \sum_{K_{1,r_K}=1}^{3} u_4^{2\,3,K_{1,r_K}} K_{4j}^{2\,3,K_{1,r_K}} + \sum_{K_{2,r_K}=1}^{3} u_5^{2\,3,K_{2,r_K}} K_{5j}^{2\,3,K_{2,r_K}} \\
&\qquad + \sum_{K_{3,r_K}=1}^{3} u_6^{2\,3,K_{3,r_K}} K_{6j}^{2\,3,K_{3,r_K}} + \sum_{K_{4,r_K}=1}^{3} u_7^{2\,3,K_{4,r_K}} K_{7j}^{2\,3,K_{4,r_K}}
\end{aligned}\right\} \qquad (f)$$

$$\left.\begin{array}{l} u_{13}^{4,1} \geq u_{13}^{4,2} \geq u_{13}^{4,3} \\ u_{14}^{4,1} \geq u_{14}^{4,2} \geq u_{14}^{4,3} \geq u_{14}^{4,4} \\ u_{12}^{1\,2,1} \geq u_{12}^{1\,2,2} \geq u_{12}^{1\,2,3} \\ u_{4}^{2\,3,1} \geq u_{4}^{2\,3,2} \geq u_{4}^{2\,3,3} \\ u_{5}^{2\,3,1} \geq u_{5}^{2\,3,2} \geq u_{5}^{2\,3,3} \\ u_{6}^{2\,3,1} \geq u_{6}^{2\,3,2} \geq u_{6}^{2\,3,3} \\ u_{7}^{2\,3,1} \geq u_{7}^{2\,3,2} \geq u_{7}^{2\,3,3} \\ u^{4} \geq \varepsilon \quad if \quad r_Y = 1,\ldots,12 \\ u^{1\,2} \geq \varepsilon \quad if \quad r_Z = 1,\ldots,11 \\ u^{2\,3} \geq \varepsilon \quad if \quad r_Z = 1,2,3 \\ v^{1}, v^{2}, p^{1\,2}, p^{1\,3}, p^{2\,3}, p^{2\,4}, p^{3\,4} \geq \varepsilon \\ u^{1\,3}, u^{2\,4}, u^{3\,4} \geq \varepsilon \end{array}\right\} (g) \tag{5}$$

Constraint (a) indicates total system efficiency, whereas constraint s (b), (c), (d), and (e) show the efficiency of the first, second, third, and fourth stages, respectively. Constraint (f) pertains to Y, Z, and K sets of variables, separating undesirable from desirable indicators and piecewise linear valuation in undesirable indicators. In other words, the K variable includes a set of 7 indicators, 3 of which are desirable: 1) total tonnage of cement produced by factory, 2) total tonnage of clinker produced by factory, and 3) the total permeation impact of drinking water and wastewater into groundwater. The other 4 indicators are as follows: 1) total produced dust particles in Mg/Nm3, 5) average annual greenhouse gas emission of NOX in Mg/Nm3, 6) average annual greenhouse gas emissions of SO2in Mg/Nm3, 7) average annual greenhouse gas emissions of CO in Mg/Nm3. These four indicators were identified as undesirable outputs with nonlinear effect, and the piecewise linear weight function was used according to expert opinion to develop specific intervals where weight variables could behave linearly. The shorter the interval, the closer the linear approximation to that variable's nonlinear effect. The valuation of variables will be nonlinear and the larger the scales of undesirable outputs, the lower their values. The concept of each equation regarding constraint (f) is as follows:

$\sum_{r_Y=1}^{14} u_{r_Y}^{4} Y_{r_Y j}^{4}$: weighted sum outputs of Y pertaining to DMU_j (j= 1, … ,42), where the number of variables is $r_Y = 1,\ldots,14$, which are the outputs resulting from Stage 4. Out of 14 variables, the first 12 are considered desirable outputs, whereas the thirteenth and fourteenth variables are undesirable. For the sake of simplicity, the weighted sum outputs was divided into Y's weighted sum desirable outputs and its weighted sum undesirable outputs. Since the piecewise linear valuation was used for modelling undesirable outputs, the weighted sum of the thirteenth undesirable output will be equal to the weighted sum of each interval specified for that output. Therefore, K_{1,r_y} in $\sum_{K_{1,rY}=1}^{3} u_{13}^{4,K_{1,rY}} Y_{13j}^{4,K_{1,rY}}$ shows the number of intervals assumed for the thirteenth Y output, $u_{13}^{4,K_{1,rY}}$ indicates the value of each K_{1,r_y} interval pertaining to the thirteenth variable, while $Y_{13j}^{4,K_{1,rY}}$ refers to the scale of the thirteenth variable in the K_{1,r_y} *th* interval. Accordingly, the scale of the thirteenth output was divided into three intervals ($K_{1,r_y} = 1,2,3$), in each of which the output indicated linear behaviour and the intervals had a different value. Moreover, K_{2,r_y} in $\sum_{K_{2,rY}=1}^{4} u_{14}^{4,K_{2,rY}} Y_{14j}^{4,K_{2,rY}}$ shows the number of intervals assumed for the fourteenth Y output, $u_{14}^{4,K_{2,rY}}$ indicates the value of each K_{2,r_y} interval pertaining to the fourteenth variable, while $Y_{14j}^{4,K_{2,rY}}$ refers to the scale of the fourteenth variable in the K_{2,r_y} th interval. Accordingly, the scale of the fourteenth output was divided into four intervals ($K_{2,r_y} = 1,2,3,4$), in each of which the output indicated linear behaviour and the intervals had a different value.

$\sum_{r_Z=1}^{12} u_{r_Z}^{1\ 2} Z_{r_Z j}^{1\ 2}$: weighted sum outputs of Z pertaining to DMU$_j$ (j= 1, … ,42), in which the number of variables is $r_Z = 1,...,12$, which are the outputs resulting from Stage 1 used as inputs for Stage 2. Out of 12 variables, the first 11 ones are considered desirable outputs, whereas the twelfth variable is undesirable. For the sake of simplicity, the weighted sum outputs were divided into Z’s weighted sum desirable outputs and its weighted sum undesirable output. Since the piecewise linear valuation was used for modelling undesirable outputs, the weighted sum of the twelfth undesirable output will equal the weighted sum of each interval specified for that output. Therefore, K_{r_Z} in $\sum_{K_{rZ}=1}^{3} u_{12}^{1\ 2,K_{rZ}} Z_{12j}^{1\ 2,K_{rZ}}$ shows the number of intervals assumed for the twelfth Y output, $u_{12}^{1\ 2,K_{rZ}}$ indicates the value of each K_{r_Z} interval pertaining to the twelfth variable, while $Z_{12j}^{1\ 2,K_{rZ}}$ refers to the scale of the twelfth variable in the K_{r_Z} *th* interval. Accordingly, the scale of the twelfth output was divided into three intervals ($K_{r_Z} = 1, 2, 3$), in each of which the output indicated linear behaviour and the intervals had a different value.

$\sum_{r_K=1}^{7} u_{r_K}^{2\ 3} K_{r_K j}^{2\ 3}$: weighted sum outputs of K pertaining to DMU$_j$ (j= 1, … ,42), where the number of variables is $r_K = 1,...,7$ showing the outputs resulting from Stage 2 used as inputs for Stage 3. Out of 7 variables, the first 3 ones are considered desirable outputs, whereas the fourth, fifth, sixth, and seventh variables are undesirable. For the sake of simplicity, the weighted sum outputs were divided into K's weighted sum desirable outputs and its weighted sum undesirable outputs. Since the piecewise linear valuation was used for modelling undesirable outputs, the weighted sum of the fourth undesirable output will equal the weighted sum of each interval specified for that output. Therefore, K_{1,r_K} in $\sum_{K_{1,r_K}=1}^{3} u_4^{2\ 3,K_{1,r_K}} K_{4j}^{2\ 3,K_{1,r_K}}$ shows the number of intervals assumed for the fourth K output, $u_4^{2\ 3,K_{1,r_K}}$ indicates the value of each K_{1,r_K} interval pertaining to the fourth variable, while $K_{4j}^{2\ 3,K_{1,r_K}}$ refers to the scale of the fourth variable in the K_{1,r_K} *th* interval. Accordingly, the scale of the fourth output was divided into three intervals ($K_{1,r_K} = 1, 2, 3$), in each of which the output indicated linear behaviour and the intervals had a different value. Moreover, K_{2,r_K} in $\sum_{K_{2,r_K}=1}^{3} u_5^{2\ 3,K_{2,r_K}} K_{5j}^{2\ 3,K_{2,r_K}}$ shows the number of intervals assumed for the fifth K output, and $u_5^{2\ 3,K_{2,r_K}}$ indicates the value of each K_{2,r_K} interval pertaining to the fifth variable. In addition, $K_{5j}^{2\ 3,K_{2,r_K}}$ shows the scale of the fifth output in the K_{2,r_K} *th* interval, and its scale was accordingly divided into three intervals ($K_{2,r_K} = 1, 2, 3$), where outputs showed linear behaviour and the intervals had a different value. Likewise, K_{3,r_K} in $\sum_{K_{3,r_K}=1}^{3} u_6^{2\ 3,K_{3,r_K}} K_{6j}^{2\ 3,K_{3,r_K}}$ shows the number of intervals considered for the sixth K output, and $u_6^{2\ 3,K_{3,r_K}}$ indicates the value of each K_{3,r_K} interval pertaining to the sixth variable. In addition, $K_{6j}^{2\ 3,K_{3,r_K}}$ shows the scale of the sixth output placed in the K_{3,r_K} th interval. Accordingly, the scale of the sixth output was divided into three intervals ($K_{3,r_K} = 1, 2, 3$) where outputs behaved linearly and intervals had a different value. Moreover, K_{4,r_K} in $\sum_{K_{4,r_K}=1}^{3} u_7^{2\ 3,K_{4,r_K}} K_{7j}^{2\ 3,K_{4,r_K}}$ shows the number of intervals considered for the seventh K output, and $u_7^{2\ 3,K_{4,r_K}}$ refers to the value of each K_{4,r_K} interval pertaining to the seventh variable. In addition, $K_{7j}^{2\ 3,K_{4,r_K}}$ shows the scale of the seventh output placed in K_{4,r_K}. Accordingly, the scale of the seventh output was divided into three intervals ($K_{4,r_K} = 1, 2, 3$) where outputs behaved linearly and intervals had a different value.

Constraint (g) shows the valuation differences pertaining to each undesirable output interval. Given the maximization objective function that allocates greater weights to larger scales of *y*, the corresponding DMU will be more efficient; hence when the DMU includes undesirable outputs and their scales are larger, the model will allocate greater weights to them, and, contrary to the expectation that the DMU having the largest scales of undesirable outputs cannot be more efficient, the DMU that produces larger scales of undesirable outputs will be more efficient. Therefore, this paper allocated different weights to different scales of undesirable outputs to analyze the direct effect of undesirable outputs on efficiency evaluation. In other words, a piecewise linear function was adopted to divide the undesirable outputs with nonlinear effect into specific intervals exhibiting linear behavior as well as the intervals with different values. Therefore, the value of the first interval was equal to or greater than the value of the second interval, and so on.

As discussed in detail, a piecewise linear function was employed to determine the intervals of undesirable outputs with nonlinear effect and nonlinear valuation in the following manner:

$$Y_{r_Y j}^{4,K} = \begin{cases} L_K & if\ K = 1 \\ L_K - L_{K-1} & if\ K = 2,\dots,K_j - 1 \\ Y_{rj} - L_{K-1} & if\ K = K_j \\ 0 & if\ K > K_j \end{cases} \quad ; \quad \begin{array}{l} Y_{r_Y j}^{4,K} \xrightarrow[r_Y=13]{K=1,\dots,3} [0,500],(500,1500],(1500,2500] \\ Y_{r_Y j}^{4,K} \xrightarrow[r_Y=14]{K=1,\dots,4} [0,200],(200,800],(800,1200],(1200,1500] \end{array} \tag{6}$$

$$Z_{r_Z j}^{1\ 2,K} = \begin{cases} L_K & if\ K = 1 \\ L_K - L_{K-1} & if\ K = 2,\dots,K_j - 1 \\ Z_{rj} - L_{K-1} & if\ K = K_j \\ 0 & if\ K > K_j \end{cases} \quad ; \quad Z_{r_Z j}^{1\ 2,K} \xrightarrow[r_Z=12]{K=1,\dots,3} [0,1.5],(1.5,2.5],(2.5,3] \tag{7}$$

$$K_{r_K j}^{2\ 3,K} = \begin{cases} L_K & if\ K = 1 \\ L_K - L_{K-1} & if\ K = 2,\dots,K_j - 1 \\ K_{rj} - L_{K-1} & if\ K = K_j \\ 0 & if\ K > K_j \end{cases} \quad ; \quad \begin{array}{l} K_{r_K j}^{2\ 3,K} \xrightarrow[r_K=4]{K=1,\dots,3} [0,400],(400,450],(450,550] \\ K_{r_K j}^{2\ 3,K} \xrightarrow[r_K=5]{K=1,\dots,3} [0,2060],(2060,2180],(2180,2220] \\ K_{r_K j}^{2\ 3,K} \xrightarrow[r_K=6]{K=1,\dots,3} [0,650],(650,680],(680,710] \\ K_{r_K j}^{2\ 3,K} \xrightarrow[r_K=7]{K=1,\dots,3} [0,126],(126,129],(129,131] \end{array} \tag{8}$$

4. Application

As discussed earlier, the proposed model was tested by collecting actual data from 42 Iranian cement companies listed on the Tehran Stock Exchange in 2016. The proposed model is able to evaluate sustainable performance in different industries. Inputs and desirable and undesirable outputs were determined for each stage. The general network structure includes 42 homogenous DMUs, all of which have four stages and also similar internal structures and internal relations shown in Figure 3. According to the indicators presented in Table 1 and the general network structure model proposed previously, undesirable outputs and their nonlinear effect were taken into account and given nonlinear values for the efficiency evaluation of companies in GAMS. Table 1 presents the results.

Table 1. Efficiency scores for 42 cement companies

DMU	input	pl-output	DMU	input	pl-output
1	1	1	22	1	1
2	1	1	23	1	1
3	1	1	24	1	0.956
4	1	1	25	1	0.995
5	1	0.999	26	1	1
6	1	1	27	0.976	0.752
7	1	1	28	1	0.983
8	1	1	29	1	1
9	1	1	30	1	0.861
10	1	1	31	1	1
11	1	1	32	1	1
12	1	1	33	1	1
13	1	1	34	1	0.931
14	1	0.937	35	1	0.948
15	1	0.949	36	1	1
16	1	0.863	37	1	1
17	1	1	38	1	0.975
18	1	0.996	39	1	1
19	1	1	40	1	1
20	1	0.890	41	1	0.939
21	1	1	42	1	0.931

The first column shows the numbers of DMUs, including 42 cement companies, and the second column shows the efficiency scores for companies according to undesirable outputs, and the linear behaviour and values of variables based on the method proposed by Tyteca (1997) was employed for efficiency measurement, which uses undesirable outputs as inputs at each stage. The results indicate that this method is unable to differentiate DMUs efficiency, and the unreliable results cannot be used for decision-making. The piecewise linear function was used in the third column to divide the undesirable outputs into different intervals and execute the model. In this case, the model aims to allocate lower values to highly undesirable outputs (if the production rate increases due to undesirable outputs, the maximization objective function of production rate will increase the amount of undesirable outputs, and since larger amounts of raw materials will be used for producing fewer desirable outputs, an increasing production rate will lead to a reduced value).

5. Conclusion

The DEA technique is employed to identify the causes of DMUs inefficiency in a competitive environment, and has been widely used in different areas. The production of undesirable outputs such as waste, pollution, aerosols, and others alongside desirable outputs is one of the main reasons for inefficient production processes. The production of undesirable outputs that have a negative impact on DMUs efficiency will be unavoidable and the presence of such outputs has a significant effect on estimation of DMUs efficiency. In such conditions, the designated efficiency evaluation method should be able to decrease undesirable outputs and increase desirable ones while being compatible with the concepts of production theory. This paper was meant to develop a DEA-based model for evaluating the sustainable performance of different industries and propose a new model according to the NDEA technique and a piecewise linear function, which considers the effects of undesirable outputs in efficiency evaluation and distinguishes the valuation between DMUs. The undesirable outputs of interest have nonlinear values; in other words, their values are equal to their inverse scales and the value of each variable was assumed to be equal to its corresponding scale. The proposed model was employed to evaluate the cement companies listed on Tehran Stock Exchange. According to the results, this method was more accurate, useful and realistic than other methods that consider variables to behave linearly.

REFERENCES

[1] **Badiezadeh, T., Saen, R. F. & Samavati, T. (2018),** ***Assessing Sustainability of Supply Chains by Double Frontier Network DEA: A Big Data Approach.*** *Computers & Operations Research*, 98, 284-290;

[2] **Charnes, A., Cooper, W. W. & Rhodes, E. (1978),** ***Measuring the Efficiency of Decision Making Units.*** *European journal of operational research*, 2(6), 429-444;

[3] **Charnes, A., Cooper, W. W., Huang, Z. M. & Sun, D. B. (1990),** ***Polyhedral Cone-ratio DEA Models with an Illustrative Application to Large Commercial Banks.*** *Journal of econometrics*, 46(1-2), 73-91;
[4] **Cook, W. D. & Zhu, J. (2009),** ***Piecewise Linear Output Measures in DEA (third revision).*** *European Journal of Operational Research*, 197(1), 312-319;
[5] **Cook, W. D., Liang, L. & Zhu, J. (2010),** ***Measuring Performance of Two-Stage Network Structures by DEA: A Review and Future Perspective.*** *Omega*, 38(6), 423-430;
[6] **Cook, W. D., Yang, F. & Zhu, J. (2009),** ***Nonlinear Inputs and Diminishing Marginal Value in DEA.*** *Journal of the Operational Research Society*, 60(11), 1567-1574;
[7] **D'Inverno, G., Carosi, L., Romano, G. & Guerrini, A. (2018),** ***Water Pollution in Wastewater Treatment Plants: An Efficiency Analysis with Undesirable Output.*** *European Journal of Operational Research*, 269(1), 24-34;
[8] **Despotis, D. K. & Sotiros, D. (2014),** ***Value-based Data Envelopment Analysis: A Piece-wise Linear Programming Approach.*** *International Journal of Multicriteria Decision Making*, 4(1), 47-68;
[9] **Despotis, D. K., Stamati, L. V. & Smirlis, Y. G. (2010),** ***Data Envelopment Analysis with Nonlinear Virtual Inputs and Outputs.*** *European Journal of Operational Research*, 202(2), 604-613;
[10] **Fare, R. & Grosskopf, S. (2000),** ***Network DEA.*** *Socio-Economic Planning Sciences*, 34(1), 35-49;
[11]**Fukuyama, H. & Weber, W. L. (2010),** ***A Slacks-based Inefficiency Measure for a Two-stage System with Bad Outputs.*** *Omega*, 38(5), 398-409;
[12] **Huang, T. H. & Chung, M. T. (2017),** ***Do Undesirables Matter on the Examination of Banking Efficiency Using Stochastic Directional Distance Functions.*** *The Quarterly Review of Economics and Finance*, 65, 194-211;
[13] **Ji, A. B., Ji, Y. & Qiao, Y. (2018),** ***DEA-Based Piecewise Linear Discriminant Analysis.*** *Computational Economics*, 51(4), 809-820;
[14] **Kalhor, A. & Matin, R. K. (2018),** ***Performance Evaluation of General Network Production Processes with Undesirable Outputs: A DEA Approach.*** *RAIRO-Operations Research*, 52(1), 17-34;
[15] **Kao, C. (2014),** ***Network Data Envelopment Analysis: A Review.*** *European journal of operational research*, 239(1), 1-16;
[16] **Kao, C. (2017),** ***Network Data Envelopment Analysis.*** *International Series in Operations Research & Management Science, Springer*, 1-16;
[17] **Kao, C. & Hwang, S. N. (2008),** ***Efficiency Decomposition in Two-stage Data Envelopment Analysis: An Application to Non-life Insurance Companies in Taiwan.*** *European journal of operational research*, 185(1), 418-429;

[18] **Khoshroo, A., Izadikhah, M. & Emrouznejad, A. (2018),** ***Improving Energy Efficiency Considering Reduction of CO2 Emission of Turnip Production: A Novel Data Envelopment Analysis Model with Undesirable Output Approach.*** *Journal of cleaner production*, 187, 605-615;

[19] **Liu, W., Zhou, Z., Ma, C., Liu, D. & Shen, W. (2015),** ***Two-stage DEA Models with Undesirable Input-intermediate-outputs.*** *Omega*, 56, 74-87;

[20] **Lotfi, F. H., Rostamy-Malkhalifeh, M. & Moghaddas, Z. (2010),** ***Modified Piecewise Linear DEA Model.*** *European journal of operational research*, 205(3), 729-733;

[21] **Lozano, S., Gutiérrez, E. & Moreno, P. (2013),** ***Network DEA Approach to Airports Performance Assessment Considering Undesirable Outputs.*** *Applied Mathematical Modelling*, 37(4), 1665-1676;

[22] **Maleki, S., Ebrahimnejad, A. & Kazemi Matin, R. (2019),** ***Pareto–Koopmans Efficiency in Two-stage Network Data Envelopment Analysis in the Presence of Undesirable Intermediate Products and Nondiscretionary Factors.*** *Expert Systems*, 36(3), e12393;

[23] **Podinovski, V. V. (2004),** ***Production Trade-offs and Weight Restrictions in Data Envelopment Analysis.*** *Journal of the Operational Research Society*, 55(12), 1311-1322;

[24] **Seiford, L. M. & Zhu, J. (2002),** ***Modeling Undesirable Factors in Efficiency Evaluation.*** *European journal of operational research*, 142(1), 16-20;

[25] **Song, M., Peng, J., Wang, J. & Dong, L. (2018),** ***Better Resource Management: An Improved Resource and Environmental Efficiency Evaluation Approach that Considers Undesirable Outputs.*** *Resources, conservation and recycling*, 128, 197-205;

[26] **Tyteca, D. (1997),** ***Linear Programming Models for the Measurement of Environmental Performance of Firms-concepts and Empirical Results.*** *Journal of productivity analysis*, 8(2), 183-197.